

width of 14.6 ps at a wavelength of 944 nm was obtained, with an optical power of 1 mW. The timing jitter was 490 fs, demonstrating that active-waveguide devices can be used in multiple-source time-division multiplexing. Grating wavelength control is very important for this application. Without wavelength control, the multiple-source TDM propagation distance in a dispersive medium is limited primarily by wavelength variations between devices, rather than the mode-locked spectral width.

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A FINITE-ELEMENT-METHOD FREQUENCY-DOMAIN APPLICATION OF THE PERFECTLY MATCHED LAYER (PML) CONCEPT

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KEY TERMS

Perfectly matched layer (PML), reflectionless absorption of waves, absorbing boundary conditions, finite-element method, frequency domain

ABSTRACT

The perfectly matched layer (PML) concept, introduced by Berenger with the aim of synthesizing an absorbing boundary condition (ABC) for the finite-difference-time-domain (FDTD) method, is modified and extended to the frequency domain for FEM applications. The governing equations that characterize this approach require neither the splitting of the field components of interest, nor do they involve negative conductivity parameters. © 1995 John Wiley & Sons, Inc.

1. INTRODUCTION

Berenger [1] has recently introduced the concept of a perfectly matched layer (PML), for reflectionless absorption of electromagnetic waves, which can be employed as an alternative to the conventional absorbing boundary conditions (ABCs) for mesh truncation in the finite-difference-time-domain (FDTD) method. Berenger's approach employs 12 curl-type equations, involving *split* components of electric and magnetic fields, that replace the six conventional curl equations. Berenger's equations, that are designed primarily for time-domain applications, are not too well-suited for finite-element (FE) formulation in the frequency domain. Other workers, namely Rappaport [2], and Chew and Weedon [3], have presented an alternative derivation based on the concept of lossy anisotropic mapping of space to obtain a perfectly matched absorbing layer for finite-difference-frequency-domain (FDFD) and FDTD applications. Unfortunately, however, these non-Maxwellian equations do not have a desirable form for FE formulation, because the weak form of the above equation involves a surface integral term that is difficult to handle. In contrast, Sacks, Kingsland, Lee, and Lee [4] have proposed a perfectly matched anisotropic absorber with negative conductivity parameters to arrive at a PML suitable for implementation in finite-element frequency- and time-domain (FEFD and FETD) applications. Mittra and Pekel [5] have shown that once the original *split-version* time-domain equations presented in [1] are cast in the frequency domain, they can be rearranged to obtain *unsplit*, but also non-Maxwellian, curl equations that contain dependent sources. The final form of these equations has been shown to be identical to the form of the *coordinate-scaled* frequency-domain equations presented in [2, 3]. In this article, the final form of the *unsplit* equations presented in [5] are further modified with a view to obtaining a system of equations that can be expressed in terms of a modified curl-like operator, which can be directly implemented in FEFD applications without the burden of a surface integral term in its weak form. Incidentally, the PML equations presented herein, which contain a modified curl operator in an *isotropic* medium, are different from those given in [4].

In the present article we present the derivation of the new equations, present some illustrative numerical results, compare these results with those obtained by using the NABC, and, finally, draw some important conclusions regarding the performance of the PML layer.

2. DERIVATION

The final form of the unsplit PML equations in the frequency domain can be written as [5]

$$j\omega\mu_0 H_x = jk_y E_z - jk_z E_y \frac{1}{1 - j(\sigma_2^*/\omega\mu_0)}, \quad (1a)$$

$$j\omega\mu_0 H_y = jk_z E_x \frac{1}{1 - j(\sigma_2^*/\omega\mu_0)} - jk_x E_z, \quad (1b)$$

$$j\omega\mu_0 H_z = jk_x E_y - jk_y E_x, \quad (1c)$$

$$j\omega\epsilon_0 E_x = jk_z H_y \frac{1}{1 - j(\sigma_2/\omega\epsilon_0)} - jk_y H_z, \quad (2a)$$

$$j\omega\epsilon_0 E_y = jk_x H_z - jk_z H_x \frac{1}{1 - j(\sigma_2/\omega\epsilon_0)}, \quad (2b)$$

$$j\omega\epsilon_0 E_z = jk_y H_x - jk_x H_y, \quad (2c)$$

where the parameters σ_2 and σ_2^* represent electric and magnetic conductivities, respectively, that also satisfy the *PML impedance matching condition*, namely, $\sigma_2/\epsilon_0 = \sigma_2^*/\mu_0$. In the above equations, it is assumed that the x - and y -directions are tangential to the interface between free space and the PML layer, and the z direction is considered to be the direction of propagation normal to the interface. At this stage, it is possible to define a new complex spatial variable $z' = z(1 - j\sigma_2/(\omega\epsilon_0)) = z(1 - j\sigma_2^*/(\omega\mu_0))$ to arrive at the *lossy mapping of space* procedure [2, 3, 5], whose governing non-Maxwellian equations can be obtained from Maxwell's curl equations in free space by replacing the Cartesian variable z with its coordinate-scaled version z' .

However, rather than trying to implement the above six equations in an FEFD formulation, one can readily manipulate these equations to get

$$(j\omega\mu_0 + \sigma_2^*)H_x = jk_y E_z - jk_z E_y, \quad (3a)$$

$$(j\omega\mu_0 + \sigma_2^*)H_y = jk_z E_x - jk_x E_z, \quad (3b)$$

$$(j\omega\mu_0 + \sigma_2^*)H_z = jk_x E_y - jk_y E_x, \quad (3c)$$

$$(j\omega\epsilon_0 + \sigma_2)E_x = jk_z H_y - jk_y H_z, \quad (4a)$$

$$(j\omega\epsilon_0 + \sigma_2)E_y = jk_x H_z - jk_z H_x, \quad (4b)$$

$$(j\omega\epsilon_0 + \sigma_2)E_z = jk_y H_x - jk_x H_y, \quad (4c)$$

where the scaled wave numbers $k_{x'}$ and $k_{y'}$ are related to the ordinary ones, namely, k_x and k_y through the equations

$$k_{x'} = k_x \left(1 - j\frac{\sigma_2}{\omega\epsilon_0}\right) = k_x \left(1 - j\frac{\sigma_2^*}{\omega\mu_0}\right), \quad (5a)$$

$$k_{y'} = k_y \left(1 - j\frac{\sigma_2}{\omega\epsilon_0}\right) = k_y \left(1 - j\frac{\sigma_2^*}{\omega\mu_0}\right). \quad (5b)$$

It can be shown that (3a)–(4c) represent a version of the original PML equations in which the transverse coordinates x and y , rather than the normal coordinate z , have been scaled to arrive at the new transverse coordinates x' and y' , defined in the following manner:

$$\begin{aligned} x' &= \frac{x}{(1 - j(\sigma_2/\omega\epsilon_0))} = \frac{x}{(1 - j(\sigma_2^*/\omega\mu_0))}, \\ y' &= \frac{y}{(1 - j(\sigma_2/\omega\epsilon_0))} = \frac{y}{(1 - j(\sigma_2^*/\omega\mu_0))}. \end{aligned} \quad (6)$$

By switching back from the spectral domain to the spatial domain, one can rewrite Equations (3a)–(4c) in the following more compact form, which is also required to eventually

obtain a weak form suitable for the FEFD formulation:

$$\begin{aligned} \nabla' \times \bar{H} &= j\omega\epsilon_0 \left(1 - j\frac{\sigma_2}{\omega\epsilon_0}\right) \bar{E}, \\ \nabla' \times \bar{E} &= -j\omega\mu_0 \left(1 - j\frac{\sigma_2^*}{\omega\mu_0}\right) \bar{H}, \end{aligned} \quad (7)$$

where the modified curl operator is defined through the modified operator ∇' as

$$\nabla' = \hat{x} \frac{\partial}{\partial x'} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z}, \quad (8)$$

reflecting the fact that the transverse coordinates x and y , and not the normal coordinate z , have been scaled in (3a) to (4c). It is clear from (6) that the new partial differential operators $\partial/\partial x'$ and $\partial/\partial y'$ are related to their conventional Cartesian counterparts through the equations

$$\frac{\partial}{\partial x'} = \left(1 - j\frac{\sigma_2}{\omega\epsilon_0}\right) \frac{\partial}{\partial x} = \left(1 - j\frac{\sigma_2^*}{\omega\mu_0}\right) \frac{\partial}{\partial x}, \quad (9a)$$

$$\frac{\partial}{\partial y'} = \left(1 - j\frac{\sigma_2}{\omega\epsilon_0}\right) \frac{\partial}{\partial y} = \left(1 - j\frac{\sigma_2^*}{\omega\mu_0}\right) \frac{\partial}{\partial y}. \quad (9b)$$

It should be emphasized, once more, that the above equations are valid for ordinary elements that lie along the free space to PML region interface, with the x and y directions as transverse, and the z direction aligned with the direction of propagation normal to the interface. For elements that lie at the edges (and not at the corners) of a given three-dimensional problem domain, where one is effectively faced with two directions of propagation, say x and z , it is necessary to modify the curl-like operator to a different form based on the modified differential operator given below:

$$\nabla' = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z}. \quad (10)$$

Equation (7) retains its validity as the governing PML equation for elements located at the edges, but not at the corners, of the three-dimensional problem domain, subject to the following two conditions: (a) the curl-like operator has to be modified as described in (10); (b) the two conductivity parameters σ_2 and σ_3 , which are associated with the z and x directions [5], respectively, must be chosen equal to each other, that is, $\sigma_2 = \sigma_3$ (and, as a consequence of the *PML impedance-matching condition*, $\sigma_2^* = \sigma_3^*$). The second condition can be enforced without a loss of generality, and leads to a considerable simplification in the governing equations.

For elements that lie at the corners of a three-dimensional problem domain, and are therefore associated with three directions of propagation, the *curl-like* operator in fact reduces to the conventional curl operator. One can then write

$$\nabla' = \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}. \quad (11)$$

As was the case with elements located at the edges of the domain, (7) remains fully valid as the governing PML equation, with the conventional curl operator effectively replacing the curl-like operator for elements located at the corners of the problem domain. It should also be noted that in this case the three conductivity parameters σ_2 , σ_3 , and σ_1 , which are associated with the z , x , and y directions [5], respectively,

have been chosen to be equal, that is, we have let $\sigma_2 = \sigma_3 = \sigma_1$ (and, hence, $\sigma_2^* = \sigma_3^* = \sigma_1^*$) without loss of generality, which leads to a simpler form for the resulting PML *corner-element* equations.

Having derived appropriate systems of partial differential equations of an essentially non-Maxwellian nature involving modified curl operators for the PML region, we can proceed next to obtain a weak form of these equations suitable for implementation in the FEFD approach. Using (7) with the isotropic right-hand sides as a starting point, we can obtain a modified curl of curl of E equation, which reads

$$\nabla' \times \left[\frac{1}{(1 - j(\sigma_2^*/\omega\mu_0))} \nabla' \times \bar{E} \right] - k_0^2 \left(1 - j \frac{\sigma_2}{\omega\epsilon_0} \right) \bar{E} = 0. \quad (12)$$

The above equation can be weighted with appropriate vectorial weighting functions $\bar{\phi}_i$, which in this work have been chosen from the set of hexahedral edge element expansion functions in accordance with the Galerkin approach, and then integrated over the volume of the PML region where this equation is to be enforced. It can be readily shown that the conventional vector differential identities (as for instance the divergence theorem) remain valid when the conventional curl operator is replaced by the modified one. With the help of these identities, one arrives at the following weak form for the PML region:

$$\begin{aligned} & \int_{V(\text{PML})} \left[\frac{1}{(1 - j(\sigma_2^*/\omega\mu_0))} \nabla' \times \bar{E} \cdot \nabla' \times \bar{\phi}_i \right. \\ & \quad \left. - k_0^2 \left(1 - j \frac{\sigma_2}{\omega\epsilon_0} \right) \bar{E} \cdot \bar{\phi}_i \right] dv \\ & = \oint_{S(\text{PML})} (j\omega\mu_0 \hat{n}' \times \bar{H}) \cdot \bar{\phi}_i ds. \end{aligned} \quad (13)$$

It should be noted that the modified outward normal unit vector $\hat{n}'$ in (13) is defined in the same manner as the modified differential operator ∇' . Therefore, the contribution of the surface integral on the right-hand side of this equation will be zero, because the *scaling* was performed on the transversal components, but not on the normal ones. Besides the fact that the medium parameters on the right-hand side of the governing PML equations are isotropic in nature, this is yet another advantage of the approach presented in this article. In contrast, in the FEFD implementation of the *lossy mapping procedure*, which utilizes scaling in the normal direction, one encounters discontinuities in the normal unit vector across the interface between free space and the PML region. These discontinuities unavoidably lead to nonzero contributions from the surface integral in question. However, it should also be pointed out that in the *transversal scaling* approach, such discontinuities do appear in the normal unit vector along internal interfaces inside the PML region between the ordinary PML elements and the corner elements. In this work, the corresponding surface integral contributions originating from such internal interfaces have been neglected.

3. NUMERICAL RESULTS

To provide numerical evidence for the effectiveness of the above-outlined PML equations acting as an absorbing bound-

ary condition (ABC), the radiation from a thin, linear dipole antenna located in free space is investigated by using a three-dimensional FEFD approach in conjunction with these PML equations. The results generated with the PML-based ABC are compared to the exact, analytical results, and also to those obtained by using numerical absorbing boundary conditions (NABCs) [6] applied directly for mesh truncation in the absence of any PML-type material. The frequency of operation is chosen to be 300 MHz, and the dipole antenna is assumed to have a triangular current distribution along its total length of $0.125 \lambda_0$ (where the free-space wavelength λ_0 is equal to 1.0 m). The FEFD approach under consideration is based on a scattered electric field formulation, and utilizes a nonuniform, but orthogonal mesh structure that comprises hexahedral, first-order edge elements with 12 edges (or unknowns) per element.

A two-dimensional front view of the specific problem geometry is shown in Figure 1, with the linear dipole antenna radiating in free space. The antenna is located at the center of the problem domain, and the total thickness of the PML-type medium surrounding the domain of interest is taken to be identical in all three directions, which leads to overall three orthogonal planes of symmetry. In this work, the surrounding layers of the PML-type medium are themselves terminated by PEC surfaces, and no ABCs, either analytical or numerical, are employed to reduce the reflections originating from the termination of the outermost layer of the PML. The two conducting parts of the antenna are modeled in the FEFD approach by using a number of consecutive edge elements filled with the PEC material, and the two parts are joined at the center of the antenna by a single edge element filled with free space to represent the gap. A known, uniform electric field is assigned to the four x -directed edges of the central *gap element* to produce a triangular current distribution along the length of the antenna. The width and depth of the antenna model are chosen to be $0.025\lambda_0$ in the y and z directions.

For the first numerical example, the size of the problem domain of interest is selected to be $0.425\lambda_0 \times 0.225\lambda_0 \times 0.225\lambda_0$ in the x , y , and z directions, respectively, excluding the surrounding PML-type medium. The total thickness of the PML region (away from the *interior region*) is chosen to be $0.3\lambda_0$ in all three directions, and is subdivided into eight layers. The orthogonal FEM mesh that extends over this

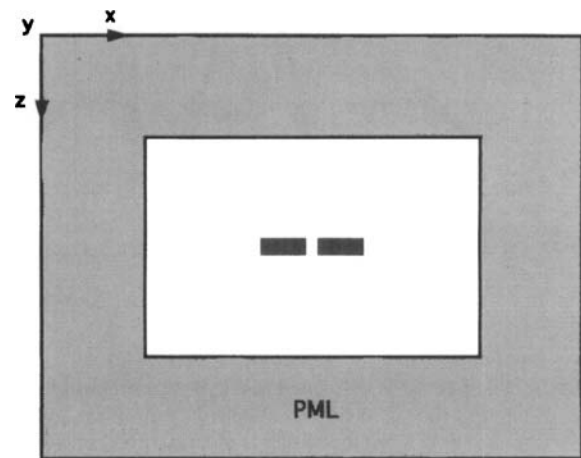


Figure 1 Linear dipole antenna radiating in free-space region surrounded by PML medium

particular problem domain gives rise to a total of 57,364 unknowns. The variation of the main field component of interest, namely, E_x , radiated by the dipole antenna in free space, and calculated by the FEFD approach in conjunction with the PML-based ABC, is given in Figure 2(a) for different profiles of the conductivity parameter σ_2 . Three different conductivity profiles have been investigated, namely, uniform profile (solid curve), linear profile (dotted curve), and quadratic profile (dashed curve). The calculated field values in all three cases are compared to the exact one determined from the analytic field expressions for an infinitely thin dipole antenna. It should be noted that all of the curves have been obtained by sampling the respective field values on a line that runs parallel to the antenna at a distance of $0.1\lambda_0$ underneath it. The exact field variation is represented by a solid line with circular markers. It can be seen that the agreement between the calculated and exact results is acceptable for all three conductivity profiles, although the numerical accuracy appears to be slightly higher for the quadratic one. This finding indicates that a PML medium made up of eight layers is sufficient in terms of its ability to simulate a reasonably reflectionless and absorptive medium, suitable for truncating a relatively small problem domain of interest. To provide an additional basis of comparison, the E_x variation was com-

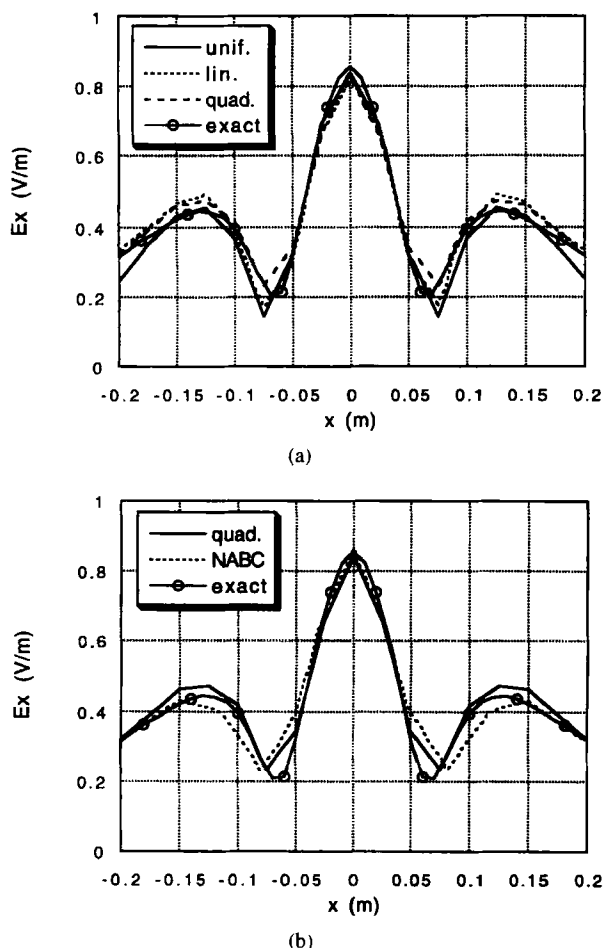


Figure 2 Comparison of calculated (using PML medium with eight layers of uniform, linear, quadratic conductivity profiles) and exact E_x at a distance of $0.1\lambda_0$ from the dipole antenna. (b) Comparison of calculated (using PML medium with eight layers of quadratic conductivity profile and NABC separately) and exact E_x at a distance of $0.1\lambda_0$ from the dipole antenna

puted by using the same FEFD approach over the same problem domain in conjunction with the *two-edge scheme* of the NABCs described in [6]. The NABC results are plotted in Figure 2(b) with a dotted curve, and are compared to the PML result for a quadratic profile with the highest numerical accuracy (solid line) and the exact result (solid line with circular markers). Note that the numerical accuracies of the calculated fields for the two techniques are essentially comparable, although the PML result appears to be slightly more accurate. Due to the absence of any PML layers in the NABC technique, however, the total number of unknowns is reduced to 20,224. However, the apparent advantage of the considerably lower number of unknowns does not translate into a proportional reduction in the CPU-time requirement, because of the nonsymmetric nature of the FEM global matrix associated with the application of the NABCs.

In the second numerical example, the size of the problem domain of interest is increased to $0.545\lambda_0 \times 0.445\lambda_0 \times 0.445\lambda_0$, not including the eight layers of the surrounding PML-type medium with a total thickness of $0.3\lambda_0$ in all three directions. The orthogonal FEM mesh that is applied on this particular problem domain generates a total number of 77,796 unknown electric field values along the element edges. Figure 3(a) shows the variations of the field component E_x , as computed by using the FEFD approach with uniform, linear, and quadratic conductivity profiles along the eight layers of the PML medium in all three directions. The computed field results are compared to the exact one, all of them being obtained along field points that lie on a line parallel to the antenna at a distance of $0.2\lambda_0$ underneath it. We observe, once again, that the accuracy of the field values obtained with the quadratic profile is slightly higher than that of the uniform and linear profiles. The performance of the PML-type medium with eight layers appears to be quite satisfactory and provides us with a guideline for synthesizing an effective ABC for the larger problem domain as well. The corresponding E_x variation computed by using a *two-edge NABC scheme* [6] is shown in Figure 3(b), together with the PML result obtained for a quadratic conductivity profile and the exact, analytical result. It is seen that the numerical accuracies of both techniques are once again essentially comparable, with the PML result being slightly more accurate.

The final numerical application deals with a problem domain whose physical dimensions are the same as those of the second case. However, in this case, the number of the surrounding PML layers is lowered to 4, with a total thickness of $0.2\lambda_0$ in all three directions, with a view to reducing the number of layers in order to ease the intensive memory requirements associated with a large number of layers of PML-type medium. The total number of unknowns is in this case given by 30,116. The computed variations of E_x for uniform, linear, and quadratic conductivity profiles in the PML region are shown and compared to the exact field behavior in Figure 4. We note that the accuracy of the results worsens, particularly for the quadratic conductivity profile, and this is attributable to the fact that the amount of attenuation provided by the PML medium simply becomes insufficient as the number of layers is reduced. Consequently, one is faced with the necessity of loosening the discretization in order to compensate for that effect, which, in turn, imposes a severe burden on the numerical accuracy of the results, because since the extremely rapid decay of the fields in the PML region cannot be accurately captured by first-order elements in a coarsely discretized mesh. Clearly, this observa-

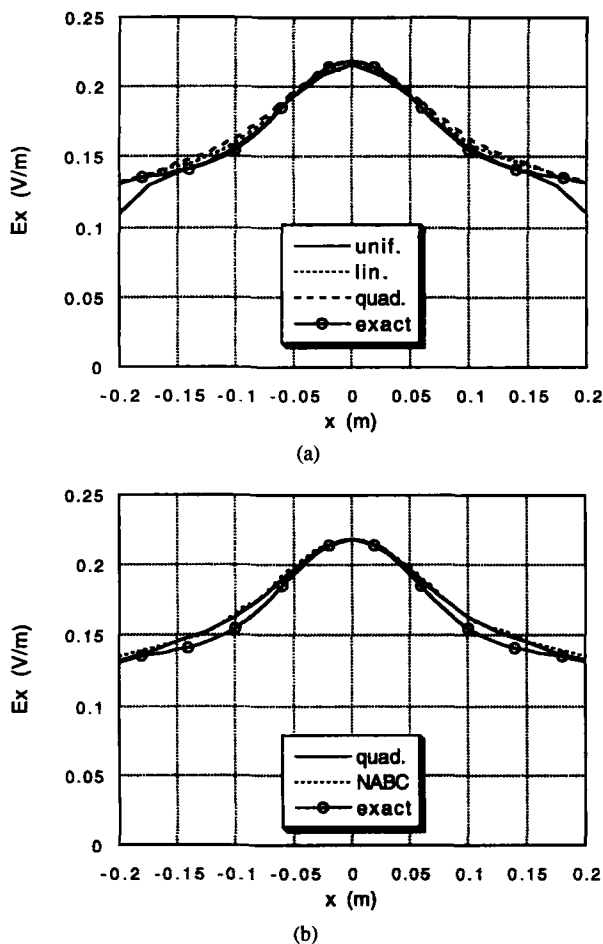


Figure 3 (a) Comparison of calculated (using PML medium with eight layers of uniform, linear, quadratic conductivity profiles) and exact E_x at a distance of $0.2\lambda_0$ from the dipole antenna. (b) Comparison of calculated (using PML medium with eight layers of quadratic conductivity profile and NABC separately) and exact E_x at a distance of $0.2\lambda_0$ from the dipole antenna

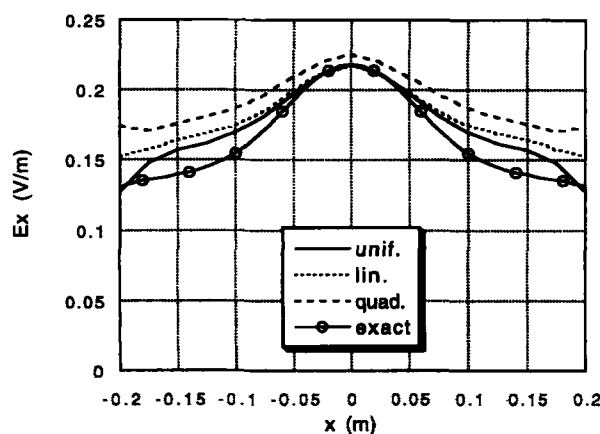


Figure 4 Comparison of calculated (using PML medium with four layers and uniform, linear, quadratic conductivity profiles) and exact E_x at a distance of $0.2\lambda_0$ from the dipole antenna

tion seems to suggest that for the PML medium with a PEC termination, a very low number of layers will not provide a satisfactory performance in FEFD applications.

4. CONCLUSIONS

In this article a new, *unsplit* version of the PML equations, suitable for implementation in finite-element-frequency-domain applications, has been presented. The governing equations of this particular version do not involve any negative conductivity parameters, and are based on a modified curl operator together with isotropic medium parameters. The version in question has been found to produce satisfactory numerical results in the context of a simple canonical problem when eight layers of the PML-type medium, with uniform, linear, or quadratic conductivity profiles, are employed to obtain an effective absorbing boundary condition. If the number of layers is reduced to save memory and CPU time, say to four layers, the PML medium terminated by a PEC wall leads to inaccurate numerical results due to a poor performance in terms of its ability to absorb of electromagnetic waves. For eight layers, however, the accuracy of the PML-based ABCs has been found to be comparable to, or slightly better than, that of the numerical ABCs (NABCs), even with a PEC termination for the outermost layer of the PML medium. It is conjectured that the accuracy of the PML approach could be increased substantially by doubling the number of layers to, say, 16, albeit at a substantial increase in computational cost in terms of CPU time and memory requirements.

It should also be mentioned that when the number of layers is only moderate, it becomes necessary to perform a considerable amount of numerical experimentation with the values of the conductivity parameter and the nature of the conductivity profile before satisfactory results can be obtained. Thus, despite the advantage of a symmetric FEM matrix of the PML approach over the NABC approach, the PML technique not only requires more CPU time and memory than the NABC, but also demands a high price for a slight increase in accuracy that is primarily achieved by using a gentle conductivity profile and a larger number of layers of the PML medium.

This has prompted us to examine the possibility of incorporating an ABC (as opposed to a PEC) termination [7] to the PML medium, because we feel that this has the potential of reducing the required total thickness of the PML medium, hence increasing the computational efficiency without sacrificing the numerical accuracy.

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SCATTERING FROM AXIALLY SLOTTED CYLINDERS: A FINITE-ELEMENT APPROACH

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KEY TERMS

Electromagnetic scattering, finite-element method, hybrid techniques

ABSTRACT

A numerical technique is presented that makes use of the finite-element method for the analysis of the scattering from perfectly conducting cylinders with cavity-backed apertures. The method of solution, based on an application of the equivalence principle, is very flexible and can be easily extended to analyze cases in which the cylinder is inserted into a complex environment. © 1995 John Wiley & Sons, Inc.

1. INTRODUCTION

As is well known, important contributions to the field scattered from complex objects may originate from cavity-backed apertures. The problem of evaluating these contributions has focused great interest in recent years and several solutions, both numerical and analytical, have been proposed in the technical literature (see, for example, References [1] and [2]). This article is concerned with the definition of a hybrid technique that is based on a particular formulation of the equivalence principle [3] and on the finite-element method (FEM) [4], for analyzing the scattering (echo width) from two-dimensional cylinders with inhomogeneously filled cavity-backed apertures. With respect to other FEM-based techniques, as, for example, the unimoment method [5] or the finite-element-absorbing-boundary-conditions technique [6], the procedure proposed allows minimization of the dimensions of the region discretized by the finite method, and is suitable to analyze more complex configurations, in which the cylinder interacts with other structures.

The aim of this communication is to demonstrate the applicability of the method proposed. This is done by refer-

ring to a simple problem: the scattering of a plane wave from an axially slotted cylinder with circular cross section. An outline of the work is described next. In particular, the problem is formulated in Section 2. A numerical solution is presented in Section 3, together with some samples of results. Finally, the features of the technique proposed are briefly discussed in Section 4.

2. FORMULATION

The geometry of the problem is depicted in Figure 1; the cavity can be arbitrarily shaped and inhomogeneously filled with media having arbitrary electromagnetic properties. The incident plane wave forms an angle ϕ^{inc} with the x axis of the reference coordinate system; ϕ denotes the observation angle with respect to the same axis. Because the main purpose of this Letter is a preliminary evaluation of the potentialities of the hybrid method proposed, in view of its application to practical three-dimensional problems, only the simplest case of polarization in which the magnetic field is parallel to the cylinder axis (TE_z case, $E_z^{inc} = 0$), will be treated. A time factor $\exp(j\omega t)$ is assumed and suppressed.

The analysis procedure consists of two consecutive steps. First, the equivalence principle is applied so as to subdivide the original problem into two separate configurations that are simpler than the original one (Figure 2). Accordingly, the aperture in the cylinder exterior surface is covered with a cylindrical perfectly conducting sheet [3]. The interior and exterior problems are coupled through the equivalent surface magnetic current densities $\mathbf{M}^{ext}$ and $\mathbf{M}^{int}$, which are impressed on opposite sides of the conducting cover of the aperture Γ_A . In particular, the exterior surface magnetic current $\mathbf{M}^{ext} = \mathbf{E} \times \hat{\rho}$ is defined at $\rho = a^+$; the interior ($\rho = a^-$) surface magnetic current has the same amplitude but opposite sign $\mathbf{M}^{int} = -\mathbf{M}^{ext}$. It is important to observe

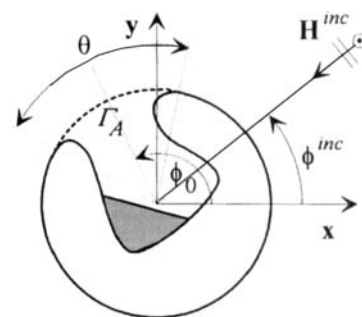


Figure 1 Geometry of the problem

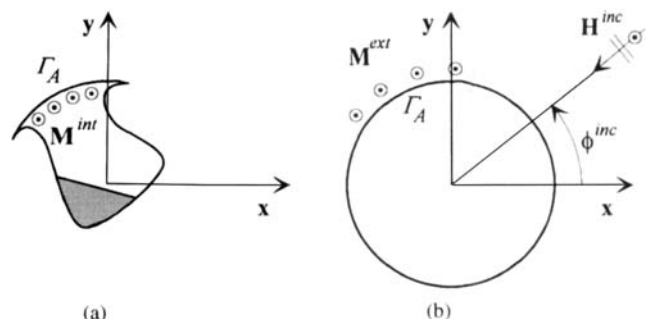


Figure 2 Subdivision of the original issue into two simpler problems: (a) interior problem; (b) exterior problem